Chapter 3.1.1

MODELLING BODY MOTION: AN APPROACH TO FUNCTIONS USING MEASURING INSTRUMENTS

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Abstract: The paper presents an approach to modelling in secondary schools where technological instruments are used for measuring and modelling motion experiences. In all cases one or more sensors measure various quantities and are connected to a calculator. In some examples we study pupils (9-th grade) who run in the class and see the Cartesian representation of their movement produced by a sensor in real time. In others, pupils (11-13-th grade) go on switchbacks or other similar merry-go-rounds and use instruments to measure some quantities (speed, acceleration, pressure), which are recorded on graphs and tables. In both cases, pupils discuss what has happened and interpret the collected data. Within a general Vygotskian frame, the authors use different complementary tools to analyse the situations: the embodied cognition by Lakoff and Núñez, the instrumental approach by Rabardel, the definition of concept by Vergnaud. In particular the role of the perceptual-motor activity in the conceptualisation of mathematics through modelling is stressed.

1. THE THEORETICAL FRAMEWORK

It is well known that pupils have difficulties in conceptualising the function concept. According to the current research, their difficulties concentrate in interpreting graphs, particularly those in which a variable is time-dependent, as for example space-time or velocity-time graphs. In fact, two main misinterpretations have been pointed out in the literature. One is the graph-as-picture interpretation, in which students expect the graph to be a picture of the phenomenon described. In kinematics, this can result in the students interpreting a graph of space versus time as if it were a road map, with the horizontal axis representing one direction of the motion rather than representing the passage of time (Clement, 1989). Another common misinterpretation is the slope/height confusion, in which students use the height of the graph at one point, when they should use the slope of the line tangent to the graph at a point, and vice-versa.

To overcome such difficulties, we have designed a teaching project where the function concept can be approached within suitable experience fields (Boero et al., 1995b) so that its meaning can be built up by students in a proper way. To this end, we use a motion sensor and a symbolic-graphic calculator, with which students create graphs and number tables to model different kinds of motion (either of their body or of other objects). The didactical aim of the teaching experiment is the construction of the concept of function as a tool for modelling motion. Our particular goal with these activities is that the students can reach competencies in describing mathematically a function, both from a global and a local point of view, starting from their perceptions and experiences with the sensor. At a more advanced level, they can use such competencies to interpret more complex situations, e.g. the motion on a switchback.

The research aim is the analysis of students' cognitive processes involved in the construction of meanings for the mathematical objects, through modelling representations. Specifically, our investigations focus on their mental dynamics while they interpret the different representations of data (tables, graphs) in order to grasp their meaning with respect to the concrete experiment of motion. This analysis is made by the observation of all the students' activities, including their gestures, language, and interactions with the instruments.

Hence our research can be framed within the challenge of Issue 1 of the Discussion Document. Specifically, it makes some contribution to the following questions:

- What are the processes of modelling? What is meant by or involved in each?
- What is the meaning and role of abstraction, formalisation and generalisation in applications and modelling?
- How much extra-mathematical context must be familiar and understood to undertake applications and modelling?

The general framework of our research is Vygotskian: the emphasis is on the social construction of knowledge and on the semiotic mediation given by cultural artefacts (Bartolini Bussi et al., 1999). The social dimension is given by the recourse to the 'mathematical discussion', orchestrated by the teacher (Bartolini Bussi, 1996); the artefacts are represented by the symbolic-graphic calculators, by the sensors and by the switchbacks.

To describe the crucial cognitive aspects of pupils' learning processes in interaction with technological instruments, we use three analysis tools:

- The embodied cognition approach by Lakoff & Núñez (2000) (see also: Arzarello, 2000a; Arzarello et al. 2003);
- The instrumental analysis by Rabardel (1995) and others (Artigue, 2001, Vérillon et al., 1995);
- The definition of concept given by G.Vergnaud (1990)[1], in particular the notion of operating invariant.

We think it is possible to integrate the instrumental approach with new results from cognitive science, in particular embodied cognition. These two approaches help us to analyse the students' activities from a new point of view. In fact, if the instrumental approach can give us a framework to analyse the use of technologies by students, in terms of schemes of use, it is not sufficient for interpreting their mental activities, especially during the conceptualisation processes. On the other hand, cognitive science is perfectly aimed to study pupils' mental activities; however, its approach to conceptualisation processes in mathematics focuses on some fundamental aspects but does not explain all of the theoretical and symbolic features of the mathematical thinking. Hence we find it useful to embed our analysis within the framework of Vergnaud's definition of concept.

2. THE TEACHING EXPERIMENTS

A main problem for students who are requested to interpret graphs or numerical tables (which model situations) regards their static features (see Kieran, 1994; Boero et al., 1995a), which risk blocking their mental dynamics, hence inhibiting a fruitful exploration (Boero et al., 1995b). In fact, to cognitively grasp the meaning of a function one needs complex dynamic activities; for example so called fictive motion (Talmy, 1996), produced when the subject interprets a graph in a dynamic and oriented way, as if it were produced by a moving trajector. Such an activity can be observed through the words and gestures of subjects (see Lakoff & Núñez, 2000, pp. 31 and 37). From this point of view it is interesting to observe how a graph is generated on the screen of a graphic calculator, which represents data on-line measured by a sensor (CBR[2]). The observer looks at a genuinely oriented generation of the points in time, which is a sensibly different experience from perceiving a graph given in a holistic way. Such a dynamic graph is easier to interpret by subjects, when compared with a static one. This is the starting point for our first working hypothesis: suitable fields of experience (see Boero et al., 1995a) where students experience real and fictive motions, can support pupils while interpreting graphs. Such a field is our "Real data in real time", where pupils live some concrete experience (e.g., running); in the meanwhile some data are relived by an on-line measurement tool and represented in real time on the screen of a graphic calculator. Successively, pupils are asked to interpret the graphs and tables on the screen, exploiting what these mean with respect to their lived experience. In the end they are asked to analyse some of their specific features and to represent them using suitable algebraic language. Our second working hypothesis is that body, language, and instruments mediate and support the transition of students from the perceptual facts to the symbolic representation, e.g. the algebraic one: in fact they can stimulate the production of an intense cognitive activity, which is marked by rich language and gesturing activity, for example with production of grounding metaphors. The purpose of our proposal is to describe the development of students' cognitive activities from bodily (e.g. perceptual, kinetic,..) to theoretical features. In such a development a crucial point is the genesis of the meaning for mathematical objects through modelling activities exploiting temporal explorations towards their just past experience and anticipating hypothesis and conjectures. Words and gestures reveal crucial insights within this activity; in particular language provides students with a fruitful cognitive activity based on their just lived kinetic and visual experiences. This genetic process allows students: (i) to produce a mathematical sense for the graphs they see on the screen and (ii) to start and support their transition to the algebraic register. For a wider discussion see the Research Forum at PME 27 (Nemirowski et al., 2003).

The teaching experiment is organised as a long-term intervention of activities during the year, each activity lasting for two-three one hour class sessions, and possibly including some open air activity, e.g. going on switchbacks in a funfair. During the sessions the students work in groups of three-four pupils and they use the tools of the activity (e.g. a measure instrument or a graphic calculator or a sheet of paper). In each activity they have to answer some questions on a working proposal form, related to the construction of the meaning of a mathematical object. The researcher, who is present during the activity has the role of observer (she records everything with a videocamera) and guides the final discussion.

3. SOME EXAMPLES OF MODELLING ACTIVITIES

3.1 Example 1

The experiment, organised by O. Robutti, consists in a sequence of activities scheduled as follows:

1. Analysing a graph and answering some questions about the points and

their co-ordinates;
2. measuring the length of objects with different tools (ruler, meter, …) and finding regularities;
3. representing data in tables or graphs using a graphic calculator (a TI92, by Texas Instruments);
4. reliving time and distance data by using a sensor of position and analysing the collected data on the graph and in the table of the calculator screen (fig.1);
5. constructing models of a phenomenon, knowing the rate of change of a quantity vs. time;
6. measuring data of a variable quantity vs. time and modelling the phenomenon.

Each activity is divided into three parts: in the first, the students (in small groups) explore a situation (using a proper tool or by paper and pencil); in the second the groups answer some written questions which ask them to use/build suitable data representations (tables, graphs) to interpret the situation in a mathematical way (within a pencil and paper or calculator environment); in the third and final part, the students participate in a class discussion, guided by a researcher.

3.2 Example 2

In our students' schools mathematics and physics are both taught by the same teacher. The idea here is to design activities within the pupils' field of experience "Real data in real time" and to use sensors to collect data on some physical quantities (speed, acceleration, pressure) while riding on a switchback or some similar machine, and then to use graphical and numerical representations to discuss the model so obtained. The goal is for pupils to enter more and more deeply into the physical concepts experienced while going on the machines, using the mediation of the mathematical model represented on the screen of the computer. The experiment is conceived with the same philosophy as that above, but requires more mathematical knowledge: in fact pupils are 2 – 3 years older than in the previous case. This part of the experiment has been designed by G. Pezzi and his equipe in Faenza. Fig.3.1.1-1 (next page) shows the sensor-kit organised to measure the physical quantities (courtesy of Texas Instruments): the kit is assembled in a bag, which can be fastened to the experimenter's body or directly to the machine. Fig. 3.1.1-3 illustrates one of the machines (the Thunder Sierra): it is a switchback with a height difference of 32.5 m, whose structure and interesting aspects are sketched in Fig.3.1.1-2.

Using pressure measures, a profile of the road has been drawn. Moreover an accelerometer has been used to record data concerning the acceleration of the coach: the diagrams (Fig. 3.1.1-4) have been obtained using the program Graphical Analysis 3.0, using the smoothing function in order to eliminate the noise from the acceleration graphics.

Figure 3.1.1-1. Sensor kit

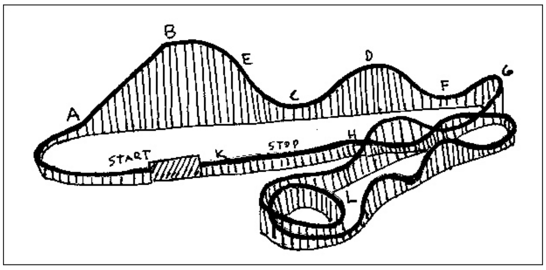

Figure 3.1.1-2. Structure of Thunder Sierra

Figure 3.1.1-3. Thunder Sierra

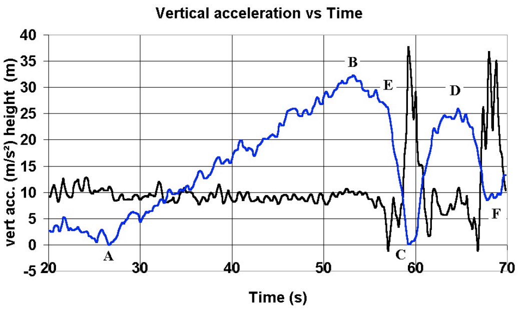

Figure 3.1.1-4. Graphical analysis

4. SOME PARTIAL CONCLUSIONS

The written protocols of all the students show that most of them have good linguistic production and a flexible co-ordination among different registers: verbal, graphical, algebraic. Moreover there is an interesting genesis of the mathematical concepts through metaphors, fictive motion and managing of the inner times (Varela, 1999; Arzarello et al., 2001). We can observe this intense cognitive activity through their gestures and linguistic productions.

It is interesting to observe that the students' cognitive activity passes through a complex evolution, which starts with their bodily experience; goes on with the evocation of the just lived experience through gestures and words; continues by connecting it with the data representation; and culminates with the use of algebraic language to write down the relationships between the quantities involved in the experiment. The recalling process has a double nature: from the one side words and gestures start the generative action function towards a suitable representation of what they have done (i.e. with tables, graphs, functions); from the other side, it allows a meaningful interiorisation of their experience. In fact, there is dialectic between mathematical concepts (for example, a function), and their representations (for example, its graph), which develops through the generative action function supported by language and gestures.

Some didactical conclusions can be drawn from our experience and may possibly be confirmed by the research, which is going on in the meanwhile. a) The approach to functions in the school often inhibits or curtails experiences that encourage the productions of fictive motions schema. For example, the graphs in books and exercises generally have a static and holistic aspect. But new technology allows teachers to design experiences where graphs can be presented in a dynamical and genetic way. b) Using grounding metaphors seems to facilitate such functions as the generative and generalising ones, which can support students in the transition to a meaningful managing of algebraic language. In fact metaphors are based on common cognitive activities that all people can do. However, grounding metaphors may be not always appreciated in the class of mathematics, since they have not a rigorous flavour. On the contrary, encouraging their production by students can facilitate the understanding of formal aspects of mathematics. As a by-product, our findings suggest that a genetic structure appears in the way metaphors are produced, which intertwines deeply with inner times of pupils. Their cognitive activity shows a continuous dynamic movement from the present to the past (their lived experience) and to the future (the hypothesis or the de-timed sentences). The analysis of connections between inner times, rhythms and metaphors reveals investigations in Mathematics Education as a promising field from the point of view of research (genesis of mathematical objects), as well as practice (which cognitive activities can the teacher encourage to facilitate pupils' understanding of mathematics?).

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[1] According to Vergnaud, a concept consists of: (i) *reference* ("l'ensemble des situations qui donnent du sens au concept"); (ii) *operating invariants* ("invariants opératoires": they allow the subject to rule the relationship between the reality and the practical and theoretical knowledge about that); (iii) *external representations* (language, gestures, symbols…).

[2] Sistema Calculator Based Ranger, Texas Instruments. For a technological description, see http://www.ti.com/calc/italia/prodotti/cbr.htm.