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Abstract: We study in this article mathematics teachers' *documentary work*: looking for resources, selecting, designing mathematical tasks, planning their succession, managing the available artifacts, etc. We consider that this documentary work is at the core of the teachers' professional activity and professional development. We introduce a distinction between available *resources,* and *documents* developed by teachers through a *documentary genesis* process, in a perspective inspired by the *instrumental approach*. Throughout their documentary work, teachers develop *documentary systems*, and the digitizing of resources entails evolutions of these systems. The approach we propose aims at seizing these evolutions, and more generally at studying teachers' professional change.

Key words: artifacts, curriculum material, digital resources, documents, documentary geneses, instruments, operational invariants, teacher beliefs, professional development.

TOWARDS NEW DOCUMENTARY SYSTEMS FOR MATHEMATICS TEACHERS?

1. Introduction

The generalized availability of digital resources for mathematics teachers entails a complete metamorphosis of the *curriculum material* (Remillard 2005), still in progress. It also yields a deep change in teachers' professional knowledge and development.

We propose here a theoretical approach aiming at illuminating the consequences of this phenomenon. However, the scope of this approach goes beyond capturing the changes brought by digital resources. We have indeed chosen, in order to analyze these changes, to consider the sets of resources used by teachers, not restricted to digital resources. We want to introduce here a general perspective for the study of teachers' professional evolution, where the researcher's attention is focused on the resources, their *appropriation* and *transformation* by the teacher or by a group of teachers working together. Similar issues have already been investigated by Adler (2000), who claims " […] in mathematics teacher education, resources in practice in context need to become a focus of attention" (p. 221). We fully agree with this claim, and mention throughout this paper many connections with Adler's conceptualization of resources. We keep nevertheless a specific interest for digital resources. This interest is not directed towards the promotion of such resources; it is meant to shed light on the use of resources as a whole, including digital and non-digital pieces, and on teachers' professional evolutions.

Studies about the integration of new technologies in the classroom have highlighted the need for holistic research approaches, suitable for capturing the wholeness of teachers' practices (Monaghan 2004). Lagrange *et al*. (2003), in their multidimensional study of research works about technology in mathematics education, already observed a growing interest for the articulation between technology and other resources. Several intertwined features of the classroom context must be considered to study integration issues. Ruthven (2008) distinguishes five such relevant dimensions:

working environment; resource system; activity format; curriculum script ("a loosely ordered model of relevant goals and actions which serves to guide [the teacher's] teaching of the topic" p.61); and time economy. We consider, as Ruthven does, that each resource must be viewed as a part of a wider *"set of resources"* (used here instead of "resource system" which suggests an a priori structure of the resources' sets).

Ruthven claims that the resources and the curriculum script interact. This perspective is close from Adler's view of resources, and from the one developed in a type of research works about teachers' use of curriculum material, that Remillard (2005) in her literature review names *"Curriculum use as participation with the text"* (p.221). The evolution of the curriculum material actually used, and the teacher's professional development are two *intertwined* processes.

Our approach is situated within a similar perspective; we want nevertheless to emphasize the following specificities:

-Examining the teachers' activity requires to consider their working environment (Cohen *et al.* 2003). We devote a particular attention to three environmental factors likely to influence teachers' work: *institutional conditions and constraints* (Chevallard 2005); *use of ICT*, and involvement in *professional collectives* (we do not analyze here collective teachers' documentary work, which deserves a specific study and belongs to our work in progress).

-We study sets of resources *not limited to curriculum material*, but including everything likely to intervene in the teachers' documentary work: discussions between teachers, orally or online; students' sheets etc. According to Adler "resources for school mathematics extend beyond basic material and human resources to include a range of other human and material resources, as well as mathematical, cultural, and social resources" (p.210). Adler proposes to distinguish between material, human, and socio-cultural resources. Cohen *et al.* (2003) also propose a comprehensive approach of resources (for the teachers and their students): "Conventional resources include teachers' formal qualifications, books, facilities, class size, and time. Personal resources include practitioners' will, skill, and knowledge. Environmental and social resources include state guidance for instruction, academic norms, professional leadership, and family support" (p.127). We do not use

the distinctions introduced by these authors; however, we also adopt a broad acceptation of resources, and consider resources entering all the categories mentioned in the above quotations.

-We devote a specific interest to the teacher's activity *outside* the class. As Ball *et al.* (2005) write it in their study of 'mathematical knowledge for teaching', teaching is not reduced to the work in class, but also includes planning, evaluating, writing assessments, discussing with parents etc. Thus we concentrate on what Remillard (2005) terms 'the curriculum mapping arena', and 'the design arena'. These arenas are naturally strongly connected with what happens in class ('the construction arena', according to Remillard), and we often mention it, but our attention is uppermost directed towards out of class activity. We focus more precisely on what we term the *teachers' documentary work:* looking for resources, selecting, designing mathematical tasks, planning their succession and the associated time management etc. This work is considered as central by several authors, in particular when a new curriculum is proposed (Christou *et al.* 2004), or in the perspective of professional development programs grounded on the work with curriculum material, or on selecting, adapting and refining mathematical tasks (Ball and Cohen 1996, Arbaugh and Brown 2006). We consider here that, even outside a particular reform or professional development program context, the documentary work is at the core of the teacher's professional activity and professional change.

-We retain a broad view of professional change. We naturally regard, as Ball and Cohen (1996) suggest, the adoption of new material as one component of professional development; but we do not curtail our study to that kind of change. We retain all kinds of professional practice evolutions. In the approach we develop, change of practice and change of professional knowledge or beliefs are connected (in a specific manner exposed in section 3), thus we consider both as simultaneous processes (like, for example, Cooney 1999, 2001). In his conceptualization of teachers' knowledge, Schulman (1986) distinguishes between subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. The documentary work obviously modifies curricular knowledge; but it can also yield evolutions of the other kinds of knowledge. In our study, we

consider teachers knowledge as a whole, without sorting out different kinds of it. This can be a further refinement of the theory, but deserves a specific study. Finally, in our analyzes of professional evolutions, we distinguish between integration of new material, other change of practice, and change of knowledge or beliefs.

We have chosen to set our study in the context of secondary school teaching because of our focus on digital resources: secondary schools are much better equipped with computers than primary schools (French Education Ministry, 2007). We have set up a series of teachers' interviews; moreover, we draw on previous research (concerning the use of e-exercises bases, and a disposal for teachers' training) for additional data. We present our methodology and these data in section 2.

A central reference for the theoretical elements we develop is the *instrumental approach* in mathematics (Guin *et al.* 2005) and the work of Rabardel (1995) grounding it. We recall the principles of this approach, and expose first steps of the generalization we propose in section 3. We introduce in particular a distinction between *resources* and *documents*, and the notion of *documentary genesis*.

Documents are developed throughout these documentary geneses. For a given teacher, these documents are organized in a *documentary system,* and the geneses are deeply interconnected with in the teachers' professional development. Analyzing the documentary system and its evolutions permits to study the professional development. We focus on these topics in section 4.

2. Data collection and methodology

We propose here the germs of a new approach of teachers' documentary work and professional development. This theoretical elaboration draws on previous theoretical elements, provided in particular by the instrumental approach; and on field data. More precisely, the data and the theoretical development we propose articulate in two directions. On one way, analyzes of the data are used in the article as examples to display and illustrate the use of the theory. On the other, these analyzes provide evidence of the theory's consistency and relevance in the realm of teachers'

documentary work; and the data also contributed to shape the concepts, by leading us to focus on particular aspects of Rabardel's theory (1995, 2005), or to introduce new refinements. These field data have three distinct origins; the two first correspond to previous research we were engaged in.

- We investigate for several years the consequences of the intervention of e-exercises bases in class (Gueudet 2006). We studied in particular the teachers' uses of a software called 'Mathenpoche' ('Maths in the pocket', shorten as MEP[1] in what follows), designed by a team of voluntary teachers, and proposing exercises covering the whole curriculum from grade 6 to 9 (Bueno-Ravel and Gueudet 2008). The teachers inscribed as 'MEP's users' can choose among the exercises to program their own MEP classroom sessions. We participated to several research projects involving MEP, in particular the GUPTEn[2] project (Lagrange *et al.* 2007) devoted to teachers' uses of ICT. In this context, we observed classroom uses of MEP, and collected descriptions of classrooms uses by teachers during three years.

- The SFoDEM (Guin and Trouche 2005) is a distance training organization for secondary school mathematics teachers, set up from 2000 to 2006 to provide continuous support for teachers in the conception, appropriation and experimentation of resources integrating ICT. It was grounded on the idea that a *collaborative work* is necessary to overcome the difficulties raised by the integration of ICT. Groups of teachers were gathered to design and experiment resources. Their work took the form of continuous communication, with one-day workshops three times a year, discussion and exchanges the rest of the time via a platform. We were thus able to follow these discussions, as well as the the evolutions of resources on this platform.

The last kind of data, central in this study, is a series of interviews. All the cases presented hereafter come from these interviews. We met nine teachers to ask them about the resources intervening in their out of class work. They received us at their homes, and the discussion took place where they usually work, next to their computer, books, binders... The one-hour interview was recorded and transcribed. The complete interview frame is presented in Appendix A. It comprises three main parts:

– A general description of everything judged useful by the teacher for her documentary work, with the identification of what is the most important;

– A detailed presentation of three selected items: a book, or a web site, or a lesson plan etc. (at least one personal production), explaining its history and use;

– A reflection about the experienced evolutions, and the expected ones (what was used 10 years ago, what do they think they will use, or what would they like to use 10 years later).

We selected the teachers interviewed according to several specifications. First, their age and professional experience: most of them are aged between 40 and 50, and have at least 10 years of professional experience (this enables them to answer to the third part of the interview). But the most important for us was to constitute a panel of teachers diverse enough with respect to the three important factors we emphasized in section 1: institutional aspects, collective involvements, use of ICT. A summary of these teachers' profiles is provided in appendix B, we give here a short description.

About institutional aspects, five of them teach in 'collège' (students from grade 6 to 9), and four in 'lycée' (students from grade 10 to 12). Moreover they have diverse institutional responsibilities, none for some of them, teachers' trainers for others, national examiners etc. Their involvements in teachers' communities vary, some of them being in particular involved in communities related with MEP, or with the SFoDEM. They also present various *degrees* of ICT integration (Assude 2008). Two of them only work with paper and pencil (degree zero), two have a scant use of ICT in class (low degree), and five have a strong degree. This last point in particular indicates clearly that these nine teachers must not be considered as representative of the French teachers at large. Because of our focus on digital resources, we selected a majority of teachers familiar with ICT use.

The data we gathered are the transcribed interviews, observations of their workplaces organization at home, of the digital files' organization on their computer, and selected materials (students' sheets, textbooks extracts, agenda etc., some on paper and some digital). We retained from the

interviews all the relevant information: list of resources mentioned, evocation of interactions with colleagues and of participation in professional development programs, and for the three resources selected as the most important, description of their use in class, of their history, evolutions, causes of evolution. Capturing the work done by a teacher out of class is difficult for material reasons. We are very grateful to the teachers who accepted to receive us at their homes. Naturally, the corresponding results are grounded on a reconstruction of their work and of their work's evolutions from their declarations and collected resources, and not on direct observations of in class or out of class. These reconstructions where naturally controlled by our classrooms observation experience, and by the knowledge we built along the years about MEP's users, and SFoDEM's participants; nevertheless, this methodological aspect induces an obvious limitation of our work. However, it also yields consequences interesting for our purpose in leading to balance in class and out of class work, and even to bring the focus a little more on the latest, which is too often neglected by researchers.

Another limitation is brought by the national character of our study. France is a developed country; the teachers we interviewed work in schools equipped with computer laboratories connected to the Internet, and most of them also have one or several computers at home. Another French feature is a strong national instructional guidance, with a national curriculum complemented by additional commentaries and advice published in 'curriculum guides'. But the textbook market is not controlled by the ministry, and a great variety of textbooks exists. The mathematics teachers' association, and the IREMs (Institutes for Research on Mathematics Teaching) are quite influential, and their publications are very popular. But collective work is still scarce, most teachers stay at school only for their courses, and go back home for their out of class work. All these national features certainly influence our results, because documentary geneses and their outcomes naturally depend on the available resources, on national culture (Leung *et al.* 2006), institutions, and collectives. We hypothesize that the concepts exposed in sections 3 and 4 are likely to illuminate documentary work in very diverse situations; however, further studies in other countries are

certainly needed to precise the national characteristics influence.

3. Resources and documents: a dialectical relationship

3.1 Distinguishing between resources and documents

We introduce here a distinction between *resources* and *documents,* extending the distinction introduced by Rabardel (1995) between *artifact* and *instrument.*

According to Rabardel, an *artifact* is a cultural and social means provided by human activity, offered to mediate another human activity. A bicycle, a computer are artifacts, and a given language is an artifact too. The subject engaged in a goal-directed activity develops an appropriation process of an artifact. Rabardel distinguishes an artifact and the *instrument,* built from the artifact by a subject throughout his/her activity. An *instrument* comes from a *process*, named *instrumental genesis*, along which the subject builds a *scheme* of utilization of the *artifact*, for a given class of situations. A *scheme,* as Vergnaud (1998) defined it from Piaget, is an *invariant organization of activity* for a given class of situations. It comprises goals and subgoals, anticipations, rules of action, of gathering information and exercising control, and possibilities of inferences. It is structured by *operational invariants,* which consist of implicit knowledge built through various contexts of utilization of the artifact. We represent this distinction by the formula:

$$Instrument = Artifact + Scheme\ of\ Utilization.$$

Instrumental geneses have a dual nature. On one hand, the subject guides the way the artifact is used and, in a sense, *shapes* the artifact: this process is called *instrumentalization*. On the other hand, the affordances and constraints of the artifact influence the subject's activity: this process is called *instrumentation*. As Noss and Hoyles (1996, p.58) note it about computers artifacts: « Far from investing the world with his vision, the computer user is mastered by his tools ».

The work of Rabardel has grounded the development, within mathematics didactics, of the instrumental approach (Guin *et al.* 2005). This theoretical framework has been used in a great

amount of research, most of it considering, as components of instrumental geneses, students as subjects, and ICT tools as artifacts.

The long term observations we realized about the uses of MEP, and within the SFoDEM project, support interpretations in terms of geneses for teachers. Teachers who integrated MEP in their practice organize more individualized teaching than they did before, because MEP offers the possibility to program different sessions for different students (instrumentation: influence of MEP's affordances on the teachers' activity). In one of the SFoDEM's groups, working on problem solving, teachers introduced digitalized students' production on the platform, a new form of sharing information (instrumentalization, appropriation of the platform). We also observed stabilization processes, regularities appearing in the teachers' activity across different contexts consistent with the development of schemes. And another consequence of this previous research was to evidence the need to take into account not only one given artifact, but a wide range of artifacts of different natures. It led us to propose a theoretical approach inspired by the instrumental approach, with distinctive features that we will detail hereafter, and a specific vocabulary.

We use the term *resources* to emphasize the variety of the artifacts we consider: a textbook, a software, a student's sheet, a discussion with a colleague etc. A resource is never isolated; it belongs to a set of resources. The subjects we study are teachers. A teacher draws on resources sets for her documentary work. A genesis process takes place, bearing what we call *a document*. The teacher builds schemes of utilization of a set of resources for the same class of situations across a variety of contexts. Adler (2000) suggests to "think of a resource as the verb *re-source*, to source again or differently" (p.207). We agree with this suggestion; the documents can be similarly thought of as the verb *document*: to support something (here the teacher's professional activity) with documents.

Let us consider for example the professional class of situations: 'propose homework about the addition of relative numbers'. For this class of situations, a given teacher gathers resources: textbooks, her own course, a previously given sheet of exercises... She chooses among these resources to constitute a list of exercises, which is given to a class. It can be then modified,

according to what happens with the students, before using it for another class during the same year, or the next year, or even later. The document develops throughout this variety of contexts. The operational invariants can be very general, like 'the homework must be extracted from the textbook', or more precisely linked with the mathematical content, like: 'the additions proposed must include the cases of mixed positive and negative numbers, and of only negative numbers' etc. These operational invariants can be inferred from the observation of invariant behaviors of the teacher for the same class of situations across different contexts. They are teacher beliefs, and both driving forces and outcomes of the teacher's activity, instrumented by a set of resources. Thus the document is much more than a list of exercises; it is *saturated* with the teachers' experience, like a word, for a given person, is saturated with sense in a Vygotskian perspective (Vygotsky 1978). The formula we retain here is:

Document = Resources + Scheme of Utilization.

Our choice of vocabulary was done to match the terminology of document management research. According to Pédauque (2006) "A document is not anything, but anything can become a document, as soon as it supplies an information, an evidence, in short, as soon as it is authoritative" (p.12, our translation).

## 3.2 Documentary genesis: an ongoing process

Figure 1 represents a *documentary genesis process*. The instrumentalization dimension conceptualizes appropriation and reshaping processes, well known by researchers studying the design and diffusion of instructional sequences: "an instructional sequence developed by one group is necessarily reshaped and transformed while others use it" (Cobb et al. 2008, p.117). The instrumentation processes conceptualizes the influence on the teacher's activity of the resources she draws on.
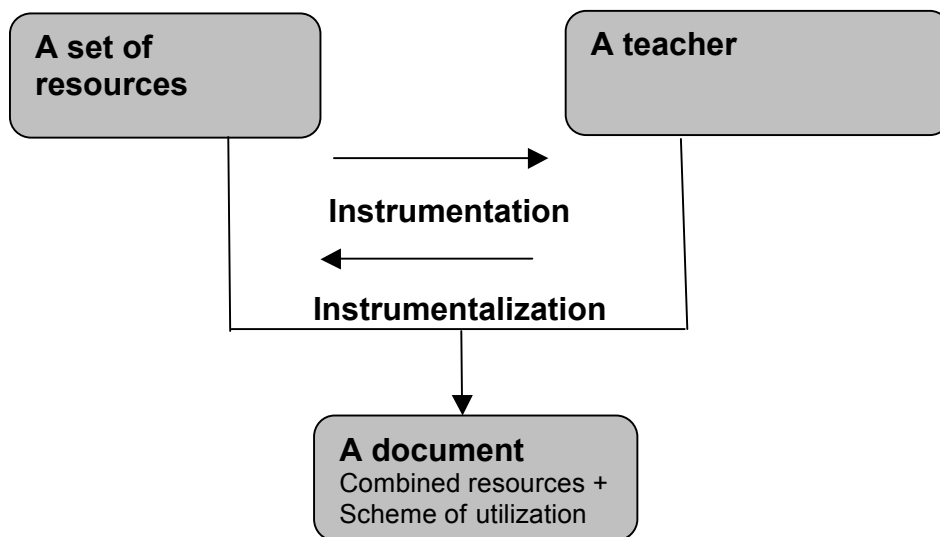
Figure 1. A documentary genesis.

Let us give a first example coming from our interviews. Frederic teaches for fifteen years to students from grade 6 to 10. During the interview, he presented us a mathematical task designed to introduce the square root in grade 9. This task concerns the square's area: several side-lengths are given, students must compute the areas, then place the points with coordinates (length, area) on a graph, draw a curve through these points, and use the curve to find approximate values of the side-length for given areas. These approximate values are then compared with the values obtained with a calculator, using the square root key. Frederic declared using this task for more than ten years (it has changed along the years, and was initially done in grade 8 before a curriculum change, but the features described above were always present). For the professional class of situations: 'Preparing the introduction of the square root in grade 9', Frederic now draws on a set of resources comprising the original textbook's extract; the student's sheet proposed the year before, with notes on it about changes he thought of when doing it in class; a slide with the points and the curve joining them, but also the students' calculators. We claim that Frederic developed a scheme of utilization of this set of resources for this class of situations. This scheme entails general operational invariants: 'a new notion must be introduced through a mathematical task evidencing the meaning of this notion'; and invariants linked with the mathematical content: 'searching for the side-length of a square for a given area evidences the meaning of the square root'; 'the square root is the reverse process of the square'; 'the calculator square root key supports the introduction of the symbol'.

The interviews analysis, and our previous research, led us to observe a very important aspect of the geneses that we want to emphasize. A documentary genesis must not be considered as a transformation with a set of resources as input, and a document as output. It is an ongoing process. Rabardel and Bourmaud (2003) claim that the design *continues in usage*. We consider here accordingly that a document developed from a set of resources provides new resources, which can be involved in a new set of resources, which will lead to a new document etc. Because of this process, we speak of a *dialectical* relationship between resources and documents. We find convenient to figure it by a helix, wrapped around an axis representing time (figure 2).
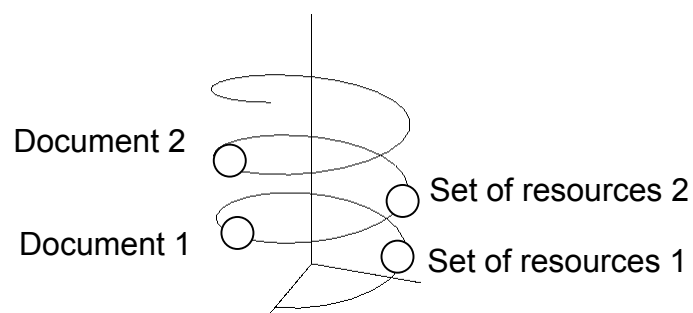
Figure 2. The resource/document dialectical relationship.

In Frederic's example, the document developed to introduce the square root a given year provides in particular an annotated text which became a resource for the following year. Let us give another example. Marie-Françoise works with students from grade 10 to 12. She organizes for them 'research narratives': problem solving sessions, where students work in groups on a problem and write down their solutions and their research processes. Thus one class of situations for Marie-Françoise is 'elaborating problems for research narratives sessions'. For this class of situations, she draws on a set of resources comprising various books and websites, ideas communicated by colleagues; but as she told us: "There is the problem and the way you enact it, because students are free to invent things, and afterwards we benefit from the richness of all these ideas, and you can build on it." We did not observe Marie-Françoise in class; however, it appears clearly in this quotation that the research narrative session depends on the students' ideas and propositions, thus the design goes on in class. But the helix represents more than the evolutions brought by enactment; the time axis invites to consider

long-term processes, along the years. Marie-Françoise collects the students' productions, she makes copies of selected productions ("most of the time, I copy, when there are interesting things in it, sometimes all of them, sometimes some of them"), and keeps digital versions of students' productions on her computer in a folder entitled 'research narratives' for each class level. Thus the document produced a given year provides new resources: a problem text, students' productions, which take place within a new set of resources that can also comprise new versions of the problem, historical texts found on a web site and related to the problem etc. And a further step in the genesis process leads to a new document for the next year, which will later bear resources for a new elaboration work.

The time axis of the helix is an important matter. The resources evolve, are modified, combined; documents develop along geneses processes and bear new resources... Long term evolutions, but also more limited events, must be taken into account.

3.3 Components, schemes and usages

We introduce here additional concepts to precise several aspects of the documentary genesis. We first expose an example, whose analysis illustrates the theoretical distinctions we propose.

Marie-Pierre is aged 40. She is teaching at secondary school for 14 years. Her school is a small "college" (150 students, in the countryside) enrolled in an experiment with technology. Teachers and students are provided with laptop computers, equipped with educational software, and USB keys. Marie-Pierre uses dynamic geometry systems, spreadsheets, MEP and other online resources. She has a digital version of the class textbook. Marie-Pierre has an interactive whiteboard in her classroom for three years, and uses it in each of her courses. She prepares a framework for her course on her laptop. She chooses extracts from web sites, and from the textbook, which will be integrated in the framework on the interactive whiteboard during the lesson. In class, she also writes on the whiteboard oral interventions from the students; sometimes the students themselves write on it. She keeps all this writing as "paperboards". She often uses these paperboards at the beginning of a new session, to recall the content of the preceding one.
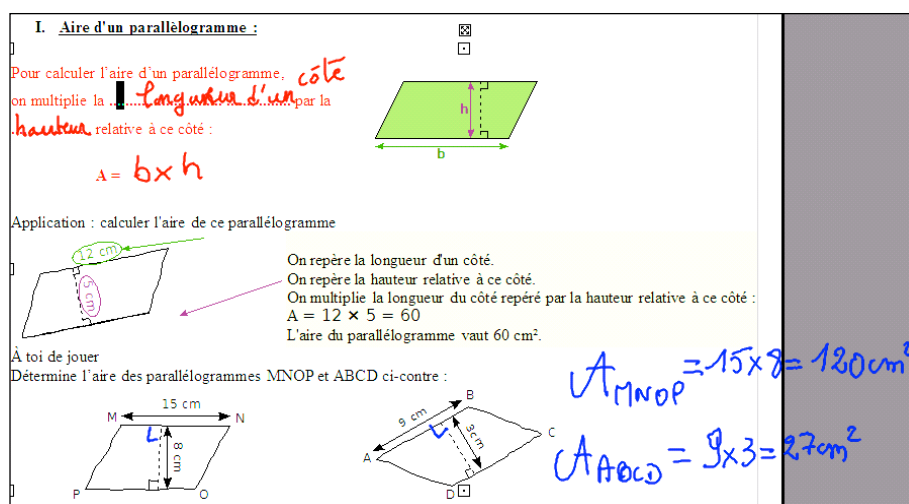
Figure 3. Marie-Pierre's case. Documentary work elements and example of interactive whiteboard display[3].

Components: material, mathematical, didactical

Considering a set of resources, or a document, requires taking into account three intertwined components:

-The material component: paper, computer, USB key, ring binder...

-The mathematical content component: notions involved, mathematical tasks and techniques;

-The didactical component: organizational elements, ranging from mapping over the whole year to planning a single one-hour session.

Let us consider, in Marie-Pierre's case, the class of situations: 'prepare a lesson about the parallelogram's area formula for grade 7' (the paperboard extract in figure 3 corresponds to that

class of situations; during the preceding session, she proposed to her students a task evidencing this formula, and the link between the parallelogram's area and the rectangle's area). She draws on a set of resources for this class of situations, and develops a document. Two aspects of the data indicate here a regularity. The first one is the paperboards; in the case of Marie-Pierre, the interactive whiteboard permits to follow through the paperboards the successive states of the board during all her sessions. For many different mathematical contents, she prepares lessons frames comprising texts with empty places, in particular when a new formula is presented. The second one is more linked with the content: Marie-Pierre declared using the same lesson for three years (most of it comes from her digital textbook).

The material component of this set of resources comprises word processing software, several web sites, the digital textbook, the interactive whiteboard, and a paper form to be filled by students. The material aspect of the document produced is a file prepared to be displayed on the whiteboard, and a paper sheet to be filled by each student. The mathematical component of this document comprises several properties, and mathematical tasks, about the parallelogram's area: the area formula, the computation of areas in several cases... A part of the organization planned for the session, belonging to the didactical component, can be observed in the document elaborated by Marie-Pierre: she will write the missing elements of the formula on the whiteboard, while the students do the same on their paper form. These missing elements will probably be determined during a class discussion. Then the students must apply the formula in direct application exercises (figure 3), and then in more complicated ones.

Scheme of utilization, and usages

A scheme of utilization of a set of resources entails an observable part, and invisible aspects. The invisible aspects are the *operational invariants*, the cognitive structure guiding the action. The observable part corresponds to the regularities in the teacher's action for the same class of situations through different contexts. This part is what we call *usages*. We distinguish between *utilizations*: when a teacher draws on a resource one, or a few times, but without developing a stable behavior

for a given class of situations; and *usages*, which correspond to a stable organization of activity, and are part of a scheme. Thus we can precise our formula by writing:

Document = Resources + Usages + Operational Invariants

In order to give elements of description of a scheme, the teacher's action should be observed on a long-term basis. The regularities identified across several events corresponding to a given class of situations permit to describe the *usages,* the *rules of action* in particular. Then the researcher can try to infer *operational invariants* from the usages. In the work we present here, as said before (section 2) we reconstructed the regularities in the teachers' documentary work from our data. Thus our analyzes in terms of scheme, must always be seen as *possible interpretations*; we propose in particular plausible operational invariants.

Let us go back to Marie-Pierre, and the class of situations: 'prepare a lesson about the parallelogram's area formula for grade 7'. We observed that she constituted a file for the interactive whiteboard, with a figure and the incomplete text: "To compute a parallelogram's area, we make the product of the.............by the associated ....". Thus the usages, for this class of situations, include the following *action rules*: 'propose a text with empty places corresponding each to only one word or expression', 'propose immediate application exercises' (general rules); 'propose a figure where the lengths intervening in the formula can be observed', 'propose different positions of the parallelogram and of the heights' (specific rules). And we infer from the data that the scheme comprises in particular the following operational invariants: 'students understand and memorize better a formula when a class discussion is organized before the teachers writes it on the board', 'immediate applications exercises support the understanding of a formula' (general operational invariants); 'the association side-horizontal and height-vertical must be avoided', 'students must precise the unit when they compute an area' (specific operational invariants).

Marie-Pierre's documentary work entails professional evolutions. One of these evolutions is naturally the integration of the interactive whiteboard. But it also led to more general practice evolutions. In particular, the interactive whiteboard led Marie-Pierre to use in class paperboards

corresponding to previous lessons, thus to make much more links with previous sessions. As Cohen *et al.* (2003) write : « Learning depends on students and teachers making bits of lessons develop and connect » (p.126). Marie-Pierre has expanded her pedagogical practice, and in particular the way she connects bits of lessons.

What we want to emphasize here is that the teacher's documentary work is strongly linked with her professional development, therefore both issues must be studied together.

4. Professional evolutions and documentary systems

4.1 The productive/constructive dialectical relationship

Adler (2000) concludes her study of resources by declaring: "Our attention shifts away from unproblematised calls for more [resources] and onto the inter-relationship between teacher and resources and how, in diverse, complex contexts and practices, mathematics teachers use the resources they have, how this changes over time, and how and with what consequences new resources are integrated into school mathematics practice" (p.221). We study here alike, not only the way teachers use the resources, but also the more or less long term evolutions yielded by these uses in their professional practice.

Rabardel (2005) introduces the *productive* and *constructive* dialectical relationship, essential for the understanding of the dynamic of teachers' professional development. The professional activity has a *productive* dimension: the outcome of the work done. But the activity also entails a modification of the subject's professional practice and beliefs, within a *constructive* dimension. Naturally, this modification influences further production processes, thus the *productive/constructive* relationship has a dialectical nature. In previous research, and in the interviews' analyzes alike, we observed that the productive dimension: design, and enactment of a session for example, was associated with evolutions of the practice, not curtailed to the integration of a new resource. We already observed this dialectical relationship at the end of the preceding section, about Marie-Pierre's case. Let us give another example.

Benoîte uses with her students mental arithmetic slide shows. She started elaborating these slide

shows two years ago, after a discussion with a colleague (in Benoîte's school, the mathematics teachers' team is used to work together). This discussion led Benoîte to a *productive* activity: the elaboration and utilization of mental arithmetic slide shows.

Figure 4. A slide used by Benoîte for mental arithmetic.

The example displayed in figure 4 is related with the class of situations 'organize mental arithmetic sessions in class about the product of decimal numbers'. The resources involved in the preparation work have material components: the slide show, a computer and an associated projection device. They also entail mathematical components: a list of products of two decimal numbers, the list of the results of the corresponding integers products'; and didactical components: the organization planned is a work with the whole class looking at the slide projected and then answering on a paper to the question, within a limited amount of time (programmed by Benoîte within the slide show). The document produced includes the resources selected; rules of action like 'prepare a precise schedule for the slides', 'propose a task of deduction of decimal numbers products from the result of the corresponding integers products'; and operational invariants like 'computing in limited time enhances the mental arithmetic procedures'; 'the students must be able to recognize and use the property: *if $a \times b = c$, then $m \times a \times n \times b = m \times n \times c$*.

Benoîte's students appreciate these slide shows; she uses them regularly, and even practices more mental arithmetic than before. Thus beyond the obvious change of practice: utilization of slide

shows in mental arithmetic, the genesis process also entails a *constructive* dimension. It induced indeed a more general change of practice: organization of more mental arithmetic. It also corresponds to a change in Benoîte's beliefs. Three years ago, she was reluctant to practice mental arithmetic in class, because of behavioral problems with the students who found it uninviting. Now she is convinced that it is possible to motivate the students, to raise their interest for mental arithmetic. Studying the evolutions of a teacher's documents contributes to the study of her professional evolution. Naturally, such a study must not be limited to the material aspect of the documents, but must investigate also the evolutions of the usages evolutions (for example, Arnaud now uses with whole classes 'help sheets' he elaborated years ago for students encountering specific difficulties) and of the operational invariants.

4.2 Documentary systems

According to Rabardel and Bourmaud (2005), the instruments developed by a subject in his/her professional activity constitute a system, whose structure corresponds to the structure of the subject's professional activity. We hypothesize here similarly that a given teacher develops a structured *documentary system*, and that this *documentary system* and the *teacher's professional practice* evolve together. From a research point of view, the observation and analysis of the documentary system permits a better understanding of the teacher's professional development, and in particular to seize the evolutions introduced by digital resources.

We do not detail the structure of a documentary systems, which deserves a specific study (in progress). We only provide here some evidence of the existence of such a structure, by analyzing Celine's case.

Celine is 36 years old, and teaches for 10 years students from grade 6 to 9. She has a computer at home, with a modem connection to the Internet, a scanner and a printer. She is equipped with mathematic software (spreadsheet, dynamic geometry software…). She uses MEP for four years, most of the time to propose technical exercises to a half class while she works with the other half class. Celine uses the official curriculum texts (downloaded on the Internet) to map her teaching over the year. She dissects this curriculum into lists of precise mathematical competences. She uses these lists to choose the mathematical content of her courses, and to program out of class MEP sessions for the preparation of assessments. Celine follows many training sessions. She buys various publications recommended by the teacher trainers. She selects in these publications ideas of mathematical tasks and designs her own tasks from these sources. She works with her colleagues only to prepare common evaluations twice a year for the grade 9 students who undertake an examination at the end of the year, and for the organization of personal monitoring for students with special needs. Celine has several ring binders for each class component, containing planning elements, courses, exercises, and assignments. In most cases, corresponding digital files can be found on her computer.

Figure 5. Celine's case. Documentary work and documentary system.

This description offers only a very incomplete view of Celine's documents. It permits nevertheless to perceive a structured organization of these documents, corresponding to the structure of the professional class of situations she encounters.

Celine uses MEP (section 2) to organize drill on technical exercises for a half class. It permits her to propose a research task on paper to the other half class, and to stay with the students in order to help them in their research processes. Thus the 'organize drill on technical exercises', and the 'organize research on rich mathematical tasks' professional situation classes are connected, as the associated documents. Celine also sets up out of class MEP sessions to help her students to prepare for the assessment at the end of the teaching of a given topic. Both situation classes 'help the students in their revisions' and 'organize the assessment' are strongly connected. Thus the corresponding documents are connected as well, in particular with similar mathematical contents. Moreover, MEP's exercises are organized along precise mathematical abilities that often coincide with Celine's dissection of the curriculum. Thus the mathematical content in MEP fits the planning she writes, because this planning is grounded on her curriculum dissections; it is thus easy for Celine to organize MEP sessions.

MEP as a resource is involved in many different documents developed by Celine, and these documents are connected with others within Celine's documentary system. This indicates the important degree of integration of MEP by Celine. Conversely, the integration of MEP by Celine depends on her possibility to associate it with other resources available for her, and in particular with resources stemming from previous documents. The integration of a new resource corresponds to a genesis process, developing a document from it and other resources. And this document must have its place within the documentary system.

We want to emphasize on this example the following essential dimensions of our approach:

- The genesis processes apply to a complex set of resources;

- They entail productive and constructive aspects;

- The reasons for the involvement of a new resource in the development of a document (we call this process the *integration of a resource in a document*) are intricate, but the study of the documentary system permits to enlighten some of these reasons.

4.3 Integrating new resources

The evolution of the resources used and of the documents developed by a teacher must be considered for different relevant time scales. The school year has naturally a specific importance: a mathematical task organized in class a given year bears resources for another year where the teacher encounters again the same class level. A shorter time can nevertheless intervene: the teaching planned for a given topic can be modified according to what happened in class. And longer periods of time can also bring important changes, like curriculum reforms, or a change of school for the teacher. Whatever the time scale, the integration and appropriation of new resources is a complex issue. A study of documentary systems and of their evolution can enlighten this issue. We illustrate it here on two examples.

Sonia (aged 50) works in a "collège" (grade 6 to 9) equipped for two years with a Virtual Learning Environment (VLE). When a student misses the class, Sonia puts her course and exercises (as pdf files) on the VLE for him/her. The student loads the files, solves the exercises and sends his/her solutions back. Sonia corrects the student's production, and sends it back. Sonia also uses the VLE to collect students' productions after spreadsheets sessions.

Celine uses MEP, and the general website hosting it. Several resources can be found on this website, in particular lists of exercises and problems called "Mathenligne" (*online mathematics*) presented as pdf files. Celine never uses these lists, she declares in the interview "I never found in it something end-to-end satisfying".

Figure 6. Sonia and Celine' case: integration or non-integration of new resources?

Sonia develops for the class of situations 'maintaining the course continuity for a missing student' a document involving several resources. On its material component, it comprises the school's VLE and Sonia's course and exercises as pdf files. The VLE is also used by Sonia to collect students' productions for correction after spreadsheets' sessions. In this case the VLE is involved in a document developed for the class of situations: 'correcting students' productions realized on computers'.

The VLE resource has been integrated by Sonia: it has been involved with other resources in genesis processes leading to a document. This document is strongly connected with others within Sonia's documentary system: her course, the mathematical tasks for spreadsheet sessions... The existence of these documents played a crucial role in the integration by Sonia of the VLE: it was easy for her to put her courses on the VLE because all of them correspond to digital files; and she was already looking for several years for ways to collect students' digital productions. The integration of the VLE contributed to develop the idea of a possible distance work with her students, which is now present in Sonia's beliefs about her teaching.

In Celine's case, several explanations of her refusal of "Mathenligne" can be proposed. She judges its mathematical content not satisfying. Moreover, the corresponding files are pdf, thus cannot be easily modified. Even the structure of the mathematical content makes a "Mathenligne sheet" difficult to modify: it proposes a mixture of technical exercises and of more complicated problems designed to be used as a whole. "Mathenligne sheets" can neither serve for drill, nor for research

tasks. They cannot combine with resources provided by Celine's documents. And they collide with Celine's beliefs about her preparation work: she prefers to design very personal productions from ideas found in diverse publications. This is a very general belief; we do not interpret it in terms of operational invariant, because it overcomes the context of a given class of situations, even a general one. Celine's interview clearly indicates that this belief shapes her documentary work.

Similar general beliefs were expressed by all the teachers we interviewed. Arnaud declares: "I have never been such a good student in mathematics, so my question has always been : what can I do to avoid my students dropping their pencils, to motivate them to do something… Maths must be fun, thus you have to go off the beaten tracks, give up textbooks and try to create things. It is what I like doing, I like to create things".

It is well known that "idealization of the professional autonomy leads to the view that good teachers do not follow textbooks, but instead make their own curriculum" (Ball and Cohen 1996 p.6). Our purpose here is not to identify this trend among the teachers we interviewed (even if such a trend certainly strongly influences the documentary work of some teachers). We want to draw attention on a kind of professional sensitivity that contributes to shape the documentary work, and the documentary system, conditioning in particular the integration or non-integration phenomena. This is certainly close to what Ruthven (2008) calls the *curriculum script* (section 1). In a more general study about professional practices, Béguin (2005) proposes the notion of '*worl*d'. A *world* is a structure piloted by professional characteristics, offering a particular cutting of the reality. It shapes a subject's point of view, his/her perception of the surrounding reality, within his/her activity in a given context. Our interviews point out expressions of sensitivity, both professional and personal, that we interpret as constituent of the teacher's world.

The discussion with Ingrid indicates such a sensitivity:

- *What would be your dream resource, ideal for your work?*

- An exercises, tasks basis. I am always looking for non-standard exercises, which is difficult to find. It must be possible to adapt the tasks, according to the students you work with. […] For a course, on the opposite, I think there can be no ideal resource. Each teacher has his own sensitivity, and tries to express it.

The worlds of the teachers we interviewed (or, at least, the part of it we were able to observe or infer) are all very different from each other. They result from their own professional, social,

personal background. Their documentary systems are very different too. And the underlying genesis processes must be taken into account, because they are part of the teacher's memory. Each teacher has a particular attachment to his/her own documentary system. A teacher's world influences the development of his/her documentary system. And the documentary system reveals features of the teacher's world.

Conclusion

We developed in this paper *a documentary approach*, focusing here on the documentary work of the teacher outside the class (even if this documentary work goes on in class). A first crucial distinction in this approach is introduced between a resource and a document, generated from a set of resources within a genesis process. It can be represented by a first equation:

Document = Resources + Scheme of Utilization.

The genesis process is permanent: the document gives birth to a new resource, likely to combine with others, to be involved in new genesis processes. We call usages the observable part of a scheme. The main part of a scheme is invisible, it corresponds to operational invariants. This can be represented by a new, more precise, equation:

Document = Resources + Usages + Operational Invariants.

Three intertwined components must be considered for the analysis of a resource, or a document: the material component, the mathematical component, and the didactical component. A document is never isolated: it belongs to a teacher's documentary system, evolving through the documentary geneses. These processes are central in the teacher's professional development, and closely related with the teacher's world.

We only presented here, and illustrated, the elementary concepts of a documentary approach. Further studies are needed to complement and precise these concepts. Proposing categories of operational invariants permitting to refine the schemes analyzes; investigating the structure of the

documentary systems for individual; examining collective documentary work; testing the approach consistency in other teaching contexts are part of our work in progress. The research perspectives naturally include the use of this approach to study teachers' documentary work, in and out of class, and the continuity and ruptures between both, but also more generally to study teachers' professional development. Documentary work appears to be central in the professional development. Thus instead of locating the main part of the teachers' professional activity in class, with the documentary work preparing this most important aspect, we consider, on the opposite, that the core of the profession is the professional development, and that the documentary work is crucial for this development.

It raises delicate methodological issues. It is necessary to observe long lasting phenomena and processes, set in different places, out of class and at the teacher's own place in particular. We consider nevertheless as fundamental for mathematics educational research to develop this documentary perspective, in particular because of the need for theoretical tools permitting to seize the current evolutions brought by the general availability of digital resources.

APPENDIX A: INTERVIEWS GUIDELINE

Teachers are interviewed at their own place, where they keep their resources. It is most of the time in a specific room, their office at home, with a computer connected to the Internet. One hour interview is planned, on a rather informal manner, but following the guidelines exposed here. The discussion is recorded, and photos of the office are taken. The interviews took place between April and June 2007, thus at the end of the school year.

First part: inventory, rationale of the documents used this year.

The questions are of the following form: "For your teaching, from the beginning of the year, which documents (book, personal document, web site...) did you use? Which has been the most important?"

Second part: detailed presentation of three documents.

We ask the teachers to present in detail three of the most important documents of the year, and their history (the teacher proposes him/herself the documents, with at least one personal production):

– If it is not a personal production: how has it been encountered, chosen, modified, used...

– If it is a personal production: which sources have been used; has it been elaborated by the teacher on her own, or with colleagues; how has it been used, has it been modified after use; what is planned with it for the future: communication to colleagues in particular.

Third part: past and future.

– Ten year ago, how would you have answered in the first part? Have you been marked by specific resources? Which ones?

– What do you think you would answer in 10 year? Which sources will you use, how would you access to these sources? Will you work for their elaboration on your own, or with colleagues? Will these documents be broadcast, and how? What would be for you a dream resource?

APPENDIX B: PROFILES OF THE TEACHERS INTERVIEWED

Explanations about the table content: Collège: grade 6 to 9, students aged 11 to 16; Lycée: grade 10 to 12, students aged 16 to 18; APM: Association of mathematics teachers; CAPES: Aptitude certificate for teaching (national competition); IREM: Institute for Research on Mathematics Teaching; INRP: National Institute for Pedagogical Research.

Teacher	Age	School	Communities Institutional responsibilities ICT degree of integration
Arnaud	47	Lycée	Teacher trainer. Degree zero.
Anaïs	57	Lycée	SFoDEM trainee, responsibilities in APM. CAPES examiner. Low degree.
Benoîte	52	Collège	Inscribed MEP's user. Low degree.
Céline	36	Collège	Trainer at the IREM, inscribed MEP's user. ICT responsibilities in the local education administration. Strong degree.
Frédéric	50	Collège	Trainer at the IREM. Degree zero.
Ingrid	27	Lycée	Trainer at the IREM, member of an INRP ICT group. Strong degree.
Marie-Pierre	40	Collège	Trainer at the IREM, APM member, inscribed MEP's user. Strong degree.
Marie-Françoise	54	Lycée	Ex-SFoDEM trainer, member of an INRP ICT group, IREM trainer. Teacher trainer. Strong degree.
Sonia	50	Collège	Member of an INRP ICT group. Former ICT responsibilities in the local education administration. Strong degree.

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[1] http://mathenpoche.sesamath.net

[2] GUPTEn holds for, in French: Genesis of Professional Use of Technologies by Teachers. This French national research project is headed by Jean-Baptiste Lagrange

[3] Translation of the screen:

1. Parallelogram's area.

To compute a parallelogram's area, we make the product of the *length of one side* by the associated *altitude.*
Application: compute the area of this parallelogram.
We identify the length of one side. We identify the corresponding altitude. We compute the product of the length of the side by the altitude : A = 12x5=60. The area of the parallelogram is 60 cm^2.

Your turn to play. Compute the area of the parallelograms MNOP and ABCD.