



# Implications for the classroom

## Research on the use of dynamic software

Keith Jones

This article sets out to summarise the research that has investigated the use of dynamic geometry software (DGS) in the teaching and learning of mathematics. This review is not intended to be exhaustive, rather the research is categorised under three main headings: *interacting with the software*, *designing teaching activities* and *learning to prove*. Overall, the research has found that DGS cannot provide a self-contained environment and that the software itself does not necessarily mean that students will learn geometry theory. Research also suggests that it can take quite a long time for the benefits of using DGS to emerge but that this investment is worthwhile in developing students' knowledge of geometry. The sorts of tasks that students tackle, the form of teacher input and the general classroom atmosphere are all important factors. To date, most research has focused on the 'classical' constructions (midpoint, perpendicular, parallel etc.) available in the DGS environment and has mostly focused on post-primary students. Little research has looked at transformations (reflection, rotation, translation, etc.) Research is only just beginning to examine the use of DGS

software in the teaching and learning of loci and the use of the macro facility available in the software.

### Dynamic geometry software

Dynamic geometry software provides a range of tools for constructing geometric objects from a range of 'primitive' objects (such as points, segments, lines, circles etc.). The tools available in the software include 'classical' constructions (midpoint, perpendicular, parallel, etc.) as well as transformations (reflect, rotate, translate, etc.) Once drawn, measurements can be taken from objects (length, angle, area etc.). The 'dynamic' aspect comes from the ability to drag defining objects, such as points, around the screen with the mouse (or tracker-ball on a laptop, or even the finger when using an interactive whiteboard). While such dragging deforms the resulting shape, some aspects remain the same. Hence the software allows a focus on the important geometrical idea of

invariance. In addition, DGS usually includes the means for drawing loci, performing animations and working with coordinates thus permitting a wide range of geometrical activity.

## Research

Over the last two decades, DGS has become one of the most widely used pieces of software in schools and colleges all over the world (there are versions of the software in French, German, Japanese, Spanish and Swedish). In terms of research, Sträßer (see page 44 for details) thinks that DGS may be one of the best, if not the best, researched type of software within mathematics education research. In respect of geometry in secondary schools, Hollebrands and colleagues consider DGS research to be the sort of software research that offers most insight into its use in the classroom.

## Interacting with the software

A number of studies have looked in depth at how students interact with the software. For example, Arzarello and colleagues have examined what they call ascending and *descending* processes shown by students using the drag mode. Ascending processes, revealed when students freely explore a situation, occur when students are looking for regularities, invariants, etc. These are moves from drawings to theory. Descending processes are moves from theory to drawings, and involve students validating or refuting conjectures, checking geometric properties, etc. These movements in the use of dragging reveal cognitive shifts from the perceptual level to the theoretical one and back again in students' mathematical activity. In the early stages of use, students in general do not use the dragging facility very much. It could be that, at first, students see dragging as something which distracts and interferes, since they are not used to seeing geometrical objects moving on paper. Once they start experimenting they begin to understand the power of the drag mode. Knowing what is invariant when a drawing is dragged is not always obvious.

## Designing teaching activities

Research on the use of DGS has highlighted how diagrams play an ambiguous role in geometry. Not only do diagrams involve theoretical objects, but

they also offer graphical – spatial properties which mean that learners may visualise things in a way that may not fit with geometrical theory. Thus teaching activities have to be carefully designed, otherwise, as Hölzl found, students may avoid mathematical analysis by looking for the practical implementation of a solution and not towards its theoretical aspects and implications, or they may circumvent the application of sophisticated tools in the software by preferring simpler ones. They may also use the tools available in a non-reflective way and/or try to deviate from the set task.

Healy and Hoyles tackle some of these problems with carefully designed sequences of tasks. By doing so they found that students could begin to connect between informal explorations and logical, deductive argument. Hadas and colleagues demonstrate how appropriate activities can be designed to create situations of contradiction for students, followed by surprise or by uncertainty and that this can lead students to seek for mathematical explanations. Laborde's experience of designing teaching scenarios based on DGS and integrating them in the regular course of classroom teaching shows that it takes a long time to reach the point where tasks genuinely take advantage of the computer environment. Finding how to manage classroom time well was also something that had to be worked on.

## Learning to prove

This area of research focuses on the vital question of whether the opportunities offered by DGS environments to 'see' mathematical properties so easily might reduce or even replace any need for proof or, on the contrary, whether such a facility might open up new ways of meaningful approaches to promoting students' understanding of the need for and the roles of proof. A range of research has examined this issue and demonstrated that judicious use of DGS can foster an understanding of proof. For example, research by Mariotti looks at how the students' view of geometry moves from an 'intuitive' one, in which geometry is seen as a collection of evident properties, to a 'theoretical' one, in which it is seen as a system of related statements that are validated by proof. According to Mariotti, this transition is greatly facilitated by the use of dynamic software that affords visualisation (a 'by eye' strategy, as she calls it), exploration and the use of problem solving strategies. The latter starts with revisiting

and manipulating drawn objects and leads to conjectures, discussion and finally to a mathematical proof. Research by Jones focuses on the evolution of students' ability to make use of precise language to arrive at an understanding of the relationships between the various properties of quadrilaterals. In this research, use of DGS clearly helped students to formulate reasonably precise statements about properties and relationships and to carry out correct deductions – both important steps in constructing proofs.

Overall, research in this area indicates that successful access to geometrical theory does not happen without carefully designed tasks, professional teacher input, and opportunities for students to conjecture, to make mistakes, to reflect, to interpret relationships among objects, and to offer tentative mathematical explanations.

## Concluding comments

A variety of research shows that interacting with DGS can help students to explore, conjecture, construct and explain geometrical relationships. It can even provide them with the basis from which to build deductive proofs. Overall, this research has found that discussions and group work in the classroom are important components. The research suggests that DGS cannot provide a self-contained environment, but that other activities are needed for students to make progress in mathematics. Indeed, classroom experiments have shown that the software itself does not grant the transition from empirical to generic objects, from the perceptive to theoretical level. The teacher plays a very important role in guiding students to theoretical thinking.

### The key messages are

1. *Dynamic geometry software used inappropriately makes no significant difference (and might make things worse);*
2. *Dynamic geometry software integrated intelligently with curriculum and pedagogy produces measurable learning gains (although it is difficult to tease out whether the gains are the direct result of using the technology or of the rethought curriculum and pedagogy, see the work of Gawlick);*
3. *What matters is **how** dynamic geometry software is used;*

4. *Using dynamic geometry software for conceptual exploration leads to conceptual gain;*
5. *Dynamic geometry software facilitates some types of learning activities, for example, exploration and visualisation, and can enhance some others, such as proof and proving.*

## Future research

As yet, little research has looked at transformations (reflection, rotation, translation, etc.) In addition, research is only just beginning to examine the use of DGS software in the teaching and learning of loci and the use of the macro facility available in the software. Furthermore, the research reviewed in this article needs careful replication and amplification. In particular, research could usefully focus on the nature of the tasks that students can tackle, the form of teacher input (and its impact) and the role of the classroom environment and culture (expectations, working methods, etc). That something works is one thing – further examples of how it can be made to work in a variety of classrooms are crucial.

As DGS evolves it will embrace more tools already associated with other forms of mathematical software. For example, a new feature in version 4 of Sketchpad is the inclusion of graph-plotting tools. This provides new opportunities and raises new research questions.



Keith Jones works at the Centre for Research in Mathematics Education at the University of Southampton, was a member of the RS/JMC inquiry into the teaching and learning of geometry and organises the geometry working group of the British Society for Research into Learning Mathematics (BSRLM), see: <http://www.soton.ac.uk/~dkj/bsrlmgeom/index.html>

### References

See separate annotated bibliography on page 44 for details of all the research mentioned in this article.

# Research Bibliographies

Every year hundreds of teachers engage in classroom-based research for a variety of purposes. Sources of support for classroom-based research include:

BECTa ICT Research Bursary up to a maximum of £10,000, see: <http://www.becta.org.uk/research/ictbursaries/guidelines.html>

DfES Best Practice Research Scholarships worth up to £2,500 each, see: [http://www.teachernet.gov.uk/Professional\\_Development/opportunities/bprs/](http://www.teachernet.gov.uk/Professional_Development/opportunities/bprs/)

DfES Professional Bursaries Scheme offers teachers in their fourth and fifth years of teaching a bursary of £500 to spend on their own professional development, see: [http://www.teachernet.gov.uk/Professional\\_Development/bursaries/](http://www.teachernet.gov.uk/Professional_Development/bursaries/)

NASUWT Teacher Research Grant Scheme of up to £3,000, see: <http://www.teachersunion.org.uk/resources/research/research.htm>

NCSL Research Associate programme offers an opportunity for school leaders (headteachers, deputies and others in team leadership roles) to engage in research, see: <http://www.ncsl.org.uk/index.cfm?pageID=randd-associates-index>

As more and more opportunities arise for teachers to get support for engaging with research, *Micromath* is devoting a section to a series of *research bibliographies* designed to provide details of the most pertinent research on using particular ICT applications in the teaching and learning of mathematics.

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## Research Bibliographies: Dynamic Geometry Software Compiled by Keith Jones

This bibliography lists research that has investigated the use of dynamic geometry software (DGS) in the teaching and learning of mathematics. The bibliography is not intended to be

exhaustive; rather it includes the major studies across the range of research that has been published.

Arzarello, F., Olivero, F., Paola, D., and Robutti, O. (2002), A Cognitive Analysis of Dragging Practises in *Cabri* Environments. *International Reviews on Mathematical Education*, 34(3), 66 – 72. Proposes a theoretical description of how students make use of the dragging facility in one DGS package.

Jones, K. (2000), Providing a Foundation for Deductive Reasoning: students' interpretations when using dynamic geometry software. *Educational Studies in Mathematics*, 44(1&2), 55-85. Shows how students' explanations can evolve from imprecise, 'everyday' expressions, through reasoning that is overtly mediated by the software environment, to mathematical explanations of the geometric situation that transcend the particular tool being used. Suggests that this latter stage should help to provide a foundation on which to build further notions of deductive reasoning in mathematics.

Gawlick, T. (2002), On Dynamic Geometry Software in the Regular Classroom. *International Reviews on Mathematical Education*, 34(3), 85 – 92. Compares classes taught using DGS to those taught without using computers. The research design adopted failed to find major differences. The students who did gain from using the software were higher achieving girls in a mixed school.

Healy L. and Hoyles C. (2001), Software tools for geometrical problem solving: potential and pitfalls, *International Journal of Computers for Mathematical Learning*, 6(3), 235-256. Reports that while DGS can be used to help students move from argumentation to logical deduction, students' understanding of the proving process is closely interconnected with how they use the software.

Hadas, N., Hershkowitz, R. and Schwarz, B. B. (2000), The Role of Contradiction and Uncertainty in Promoting the Need To Prove in Dynamic Geometry Environments. *Educational Studies in Mathematics*, 44(1&2), 127-150. Shows how, with carefully designed activities, students can engage in the construction and evaluation of mathematics arguments in which they have to use their geometrical knowledge to explain contradictions and overcome uncertainty.

Hölzl, R. (1996): How does 'dragging' affect the learning of geometry? *International Journal of Computers for Mathematical Learning*, 1(2), 169-187.

Hölzl, R. (2001): Using DGS to add Contrast to Geometric Situations – A Case Study, *International Journal of Computers for Mathematical Learning*, 6(1), 63-86.

Two articles that provide interesting insights into how students use DGS.

Laborde, C. (2001), Integration of Technology in the Design of Geometry Tasks with *Cabri-Geometry*. *International Journal of Computers for Mathematical Learning*, 6(3), 283-317.

Relates how long it can take to fully utilise the potential of DGS in the classroom.

Mariotti, M.A. (2000), Introduction to Proof: the mediation of a dynamic software environment. *Educational Studies in Mathematics*, 44(1&2), 25-53.

Explores the various influences of DGS on student learning of geometry, especially the characteristics which make it possible to introduce students to theoretical thinking. Finds that this latter process is neither simple nor spontaneous.

Sträßer, R. (2001), Cabri-Geometre: Does Dynamic Geometry Software (DGS) Change Geometry and Its Teaching and Learning? *International Journal of Computers for Mathematical Learning*, 6(3), 319-333.

Explores how, with DGS, geometrical objects are transformed into computational objects. Concludes that this deeply changes geometry if geometry is taken as a human activity integrating the use of tools such as DGS.

## Other useful books and reports

Hollebrands, K., Laborde, C., & Sträßer, R. (in preparation), The Learning of Geometry with Technology at the Secondary Level. In M.K. Heid & G. Blume (Eds.), *Handbook of Research on Technology in the Learning and Teaching of Mathematics: Mathematics Curriculum Development and Tool Development*. Greenwich/CT: Information Age.

Should provide a good overview of research on learning geometry with ICT.

King, J. R. and Schattschneider, D. (Eds) (1997), *Geometry turned on! Dynamic software in learning, teaching, and research*.

Washington: Mathematical Association of America.

Gives many examples of the ways in which DGS can be used, and some of the effects it can have.

NCET (1996), *Dynamic Geometry*. Warwick: NCET.

Discusses and gives examples of classroom use of DGS. Two sections of the report are available online at:

<http://curriculum.becta.org.uk/docserver.php?docid=1762>

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